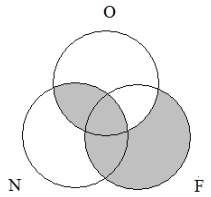
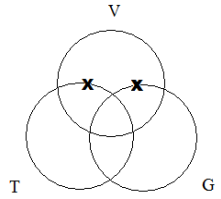
**2001b practice exam solutions**

**A)**

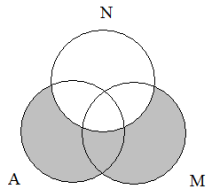
**1**
***Answer:*** Valid. Let F = *fast-food items*, O = *overpriced objects*, and N = *nutritious products.*



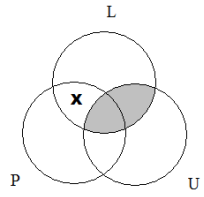

**2.**
***Answer:*** Invalid. Let V = *vegetables*, T = *tasty foods*, and G = *green foods.*



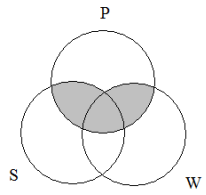

**3.**
***Answer:*** Invalid. Let M = *mechanical objects*, N = *noisy objects*, and A = *airplanes*



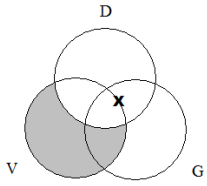
**4.**
***Answer:*** Valid. Let P = *pens*, U = *useful tools*, and L = *leaky writing implements.*



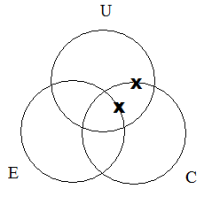

**5.** No septic tanks are swimming pools. No sewers are swimming pools. Therefore, no septic tanks are sewers.
***Answer:*** Invalid. Let S = *septic tanks*, P = *swimming pools*, and W = *sewers.*



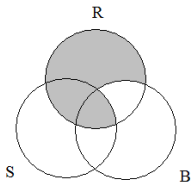

**6.**
***Answer:*** Invalid. Let V = *voice messages*, D = *distracting pieces of information*, and *G = games people play*.




**7.**
***Answer:*** Invalid. Let U = *universities*, E = *expensive places to attend*, and C = *conveniently located complexes.*



**8.**
***Answer:*** Invalid. Let S = *sports fanatics*, R = *rational creatures*, and B = *benevolent people*.




b)

1) L- it's leap year; S- someone born on a strange day; R- have reason to stay calm

L ⊃ S

(S v ~L) ⊃ R

~R

2) F- we will be fine; T- Taylor Swift releases another song;

F v T

(T ⊃ ~F) • ~T

F

3) A- everything is amusing; T- everything is tragic; W- life is weird

A • T

W

W ≡ (A v T)

4) L- logic class is fun; A- need to ask if it's fun; F- fun is important

L ⊃ ~A

~A • ~F

L • F

C)

1) S- boy is holding a shovel; B- butterfly nearby; R- 3 roses nearby
(S • B) ⊃ R
Conditional
False

2) P- package changing hands; D- dog nearby; S- snow on the ground; T – tree nearby
(P ⊃ D) v (S ⊃ ~T)
Disjunction
True

3) H- man is wearing a hat; S- man is wearing a scarf; G- man is wearing glasses; E – eggs falling out of his bucket
(H v ~S) • (~G ⊃ ~E)
Conjunction
True

d)

1. (~R ∨ S) • (~ T ⊃ R)

| R | S | T | (~ | R | ∨ | S) | • | (~ | T | ⊃ | R) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 | 1 | 1 | **1** | 0 | 1 | 1 | 1 | |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | **1** | 1 | 0 | 1 | 1 | |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | **0** | 0 | 1 | 1 | 1 | |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | **0** | 1 | 0 | 1 | 1 | |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | **1** | 0 | 1 | 1 | 0 | |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | **0** | 1 | 0 | 0 | 0 | |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | **1** | 0 | 1 | 1 | 0 | |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | **0** | 1 | 0 | 0 | 0 | |
| | | | | | | | | | | | | |

2. [U ⊃ (V ⊃ W)] • (V ∨ W)

| U | V | W | [U | ⊃ | (V | ⊃ | W)] | • | (V | ∨ | W) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | **1** | 1 | 1 | 1 | |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | **0** | 1 | 1 | 0 | |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | **1** | 0 | 1 | 1 | |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | **0** | 0 | 0 | 0 | |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | **1** | 1 | 1 | 1 | |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | **1** | 1 | 1 | 0 | |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | **1** | 0 | 1 | 1 | |
| 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | **0** | 0 | 0 | 0 | |

3. [~ X ≡ (Y•Z)] ⊃ (X ∨ Z)

| X | Y | Z | [~ | X | ≡ | (Y | • | Z)] | ⊃ | (X | ∨ | Z) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | **1** | 1 | 1 | 1 | |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | **1** | 1 | 1 | 0 | |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | **1** | 1 | 1 | 1 | |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | **1** | 1 | 1 | 0 | |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | **1** | 0 | 1 | 1 | |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | **1** | 0 | 0 | 0 | |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | **1** | 0 | 1 | 1 | |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | **1** | 0 | 0 | 0 | |

E)

1.

| | | |
|---|---|---|
| 1. Q ⊃ (~R ⊃ S) | | |
| 2. T ∨ Q | | |
| 3. ~T | | |
| 4. R ⊃ T | / S | |
| 5. Q | | 2, 3, DS |
| 6. ~R ⊃ S | | 1, 5, MP |
| 7. ~R | | 3, 4, MT |
| 8. S | | 6, 7, MP |

2.

| | | |
|---|---|---|
| 1. V ⊃ (W ∨ U) | | |
| 2. X ∨ V | | |
| 3. X ⊃ Y | | |
| 4. ~Y | | |
| 5. ~Y ⊃ ~W | / U | |
| 6. ~X | | 3, 4, MT |
| 7. V | | 2, 6, DS |
| 8. W ∨ U | | 1, 7, MP |
| 9. ~W | | 4, 5, MP |
| 10. U | | 8, 9, DS |

3.

| | | |
|---|---|---|
| 1. A ⊃ B | | |
| 2. B ⊃ (C ⊃ D) | | |
| 3. E ∨ C | | |
| 4. E ⊃ F | | |
| 5. ~F | | |
| 6. C ⊃ A | / D | |
| 7. ~E | | 4, 5, MT |
| 8. C | | 3, 7, DS |
| 9. A | | 6, 8, MP |
| 10. A ⊃ (C ⊃ D) | | 1, 2, HS |
| 11. C ⊃ D | | 9, 10, MP |
| 12. D | | 8, 11, MP |

4.

| | | |
|---|---|---|
| 1. D ∨ E | | |
| 2. D ⊃ F | | |
| 3. ~F • G | / (E ∨ H) • ~F | |
| 4. ~F | | 3, Simp |
| 5. ~D | | 2, 4, MT |
| 6. E | | 1, 5, DS |
| 7. E ∨ H | | 6, Add |
| 8. (E ∨ H) • ~F | | 4, 7, Conj |

5.

| | | |
|---|---|---|
| 1. P ⊃ (Q ⊃ ~U) | | |
| 2. R ⊃ (Q ⊃ S) | | |
| 3. (P ∨ R) • T | | |
| 4.~(Q ⊃ ~U) | | |
| 5. Q | / S ∨ ~U | |
| 6. P ∨ R | 3, Simp | |
| 7. (Q ⊃ ~U) ∨ (Q ⊃ S) | 1, 2, 6, CD | |
| 8. Q ⊃ S | 7, 4, DS | |
| 9. S | 8, 5, MP | |
| 10. S ∨ ~U | 9, Add | |

F)

1.

| | | |
|---|---|---|
| 1. ~D • ~E | | |
| 2. (D ∨ F) ∨ E | / F | |
| 3. ~(D ∨ E) | 1, DM | |
| 4. D ∨ (F ∨ E) | 2, Assoc | |
| 5. D ∨ (E ∨ F) | 4, Com | |
| 6. (D ∨ E) ∨ F | 5, Assoc | |
| 7. F | 6, 3, DS | |

2.

| | | |
|---|---|---|
| 1. I • {~[J • (K ∨ L)] • M} | | |
| 2. (~J ∨ ~L) ⊃ N | / N | |
| 3. {I • ~[J • (K ∨ L)]} • M | 1, Assoc | |
| 4. I • ~[J•∨ (K ∨ L)] | 3, Simp | |
| 5. ~[J • (K ∨ L)] • I | 4, Com | |
| 6. ~[J • (K ∨ L)] | 5, Simp | |
| 7. ~[(J • K) ∨ (J • L)] | 6, Dist | |
| 8. ~(J • K) • ~(J • L) | 7, DM | |
| 9. ~(J • L) • ~(J • K) | 8, Com | |
| 10. ~(J • L) | 9, Simp | |
| 11. ~J ∨ ~L | 10, DM | |
| 12. N | 2, 11, MP | |

3.

| | | |
|---|---|---|
| 1. [O ∨ (P • Q)] ⊃ R | | |
| 2. R ⊃ ~S | | |
| 3. P • S | / ~Q | |
| 4. S • P | 3, Com | |
| 5. S | 4, Simp | |
| 6. ~ ~S | 5, DN | |
| 7. ~R | 2, 6, MT | |
| 8. ~[O ∨ (P • Q)] | 1, 7, MT | |
| 9. ~O • ~(P • Q) | 8, DM | |
| 10. ~(P • Q) • ~O | 9, Com | |
| 11. ~(P • Q) | 10, Simp | |
| 12. ~P ∨ ~Q | 11, DM | |
| 13. P | 3, Simp | |
| 14. ~ ~P | 13, DN | |
| 15. ~Q | 12, 14, DS | |

4.

| | | |
|---|---|---|
| 1. | (S ∨ C) ∨ (I ∨ N) | |
| 2. | (S ∨ C) ⊃ U | |
| 3. | I ⊃ C | |
| 4. | ~U | / ~(U ∨ ~N) |
| 5. | ~(S ∨ C) | 2, 4, MT |
| 6. | ~S • ~C | 5, DM |
| 7. | ~C • ~S | 6, Com |
| 8. | ~C | 7, Simp |
| 9. | ~I | 3, 8, MT |
| 10. | I ∨ N | 1, 5, DS |
| 11. | N | 10, 9, DS |
| 12. | ~U • N | 4, 11, Conj |
| 13. | ~U • ~ ~N | 12, DN |
| 14. | ~(U ∨ ~N) | 13, DM |

5.

| | | |
|---|---|---|
| 1. | Q ⊃ R | |
| 2. | R ⊃ (S ⊃ T) | / ~T ⊃ (S ⊃ ~Q) |
| 3. | Q ⊃ (S ⊃ T) | 1, 2, HS |
| 4. | (Q • S) ⊃ T | 3, Exp |
| 5. | ~(Q • S) ∨ T | 4, Impl |
| 6. | (~Q ∨ ~S) ∨ T | 5, DM |
| 7. | (~S ∨ ~Q) ∨ T | 6, Com |
| 8. | T ∨ (~S ∨ ~Q) | 7, Com |
| 9. | ~ ~T ∨ (~S ∨ ~Q) | 8, DN |
| 10. | ~T ⊃ (~S ∨~Q) | 9, Impl |
| 11. | ~T ⊃ (S ⊃ ~Q) | 10, Impl |

6.

| | | |
|---|---|---|
| 1. | (P ≡ Q) ∨ P | / P ∨ ~Q |
| 2. | [(P ⊃ Q) • (Q ⊃ P)] ∨ P | 1, Equiv |
| 3. | P ∨ [(P ⊃ Q) • (Q ⊃ P)] | 2, Com |
| 4. | [P ∨ (P ⊃ Q)] • [P ∨ (Q ⊃ P)] | 3, Dist |
| 5. | [P ∨ (Q ⊃ P)] • [P ∨ (P ⊃ Q)] | 4, Com |
| 6. | P ∨ (Q ⊃ P) | 5, Simp |
| 7. | P ∨ (~Q ∨ P) | 6, Impl |
| 8. | P ∨ (P ∨ ~Q) | 7, Com |
| 9. | (P ∨ P) ∨ ~Q | 8, Assoc |
| 10. | P ∨ ~Q | 9, Taut |

G)

| | | |
|---|---|---|
| 1. | 1. Q ⊃ (~R • S) | / R ⊃ ~Q |
| | \| 2. Q | ACP |
| | \| 3. ~R • S | 1, 2, MP |
| | \| 4. ~R | 3, Simp |
| | 5. Q ⊃ ~R | 2–4, CP |
| | 6. ~ ~R ⊃ ~Q | 5, Cont |
| | 7. R ⊃ ~Q | 6, DN |

| | | |
|---|---|---|
| 2 . | 1. ~M ∨ N | |
| | 2. P | / (M ∨ ~P) ⊃ (O ∨ N) |
| | \| 3. M ∨ ~P | ACP |
| | \| 4. ~P ∨ M | 3, Com |
| | \| 5. P ⊃ M | 4, Impl |
| | \| 6. M ⊃ N | 1, Impl |
| | \| 7. P ⊃ N | 5, 6, HS |
| | \| 8. N | 2, 7, MP |
| | \| 9. N ∨ O | 8, Add |
| | \| 10. O ∨ N | 9, Com |
| | 11. (M ∨ ~P) ⊃ (O ∨ N) | 3–10, CP |

| | | |
|---|---|---|
| 3. | 1. E ⊃ ~(F ⊃ G) | |
| | 2. F ⊃ (E • H) | / E ≡ F |
| | \| 3. E | ACP |
| | \| 4. ~(F ⊃ G) | 1, 3, MP |
| | \| 5. ~(~F ∨ G) | 4, Impl |
| | \| 6. ~ ~F • ~G | 5, DM |
| | \| 7. F • G | 6, DN |
| | \| 8. F | 7, Simp |
| | 9. E ⊃ F | 3–8, CP |
| | \| 10. F | ACP |
| | \| 11. E • H | 2, 10, MP |
| | \| 12. E | 11, Simp |
| | 13. F ⊃ E | 10–12, CP |
| | 14. (E ⊃ F) • (F ⊃ E) | 9, 13, Conj |
| | 15. E ≡ F | 14, Equiv |

4.

| | | |
|---|---|---|
| 1. R ⊃ (S ∨ W) | | |
| 2. R ⊃ (T ∨ W) | | |
| 3. ~(W ∨ X) | | / R ⊃ (S • T) |
| | 4. R | ACP |
| | 5. S ∨ W | 1, 4, MP |
| | 6. T ∨ W | 2, 4, MP |
| | 7. ~W • ~X | 3, DM |
| | 8. ~W | 7, Simp |
| | 9. W ∨ S | 5, Com |
| | 10. S | 9, 8, DS |
| | 11. W ∨ T | 6, Com |
| | 12. T | 11, 8, DS |
| | 13. S • T | 10, 12, Conj |
| 14. R ⊃ (S • T) | | 4–13, CP |

QED

5.

| | | | |
|---|---|---|---|
| 1. A ⊃ [(D ∨ B) ⊃ C] | | | / A ⊃ (D ⊃ C) |
| | 2. A | | ACP |
| | | 3. D | ACP |
| | | 4. (D ∨ B) ⊃ C | 1, 2, MP |
| | | 5. D ∨ B | 3, Add |
| | | 6. C | 4, 5, MP |
| | 7. D ⊃ C | | 3–6, CP |
| 8. A ⊃ (D ⊃ C) | | | 2–7, CP |

QED

6.

| | | |
|---|---|---|
| 1. M ⊃ L | | |
| 2. ~(K • N) ⊃ (M ∨ L) | | / K ∨ L |
| | 3. ~(K ∨ L) | AIP |
| | 4. ~K • ~L | 3, DM |
| | 5. ~L • ~K | 4, Com |
| | 6. ~L | 5, Simp |
| | 7. ~M | 1, 6, MT |
| | 8. ~K | 4, Simp |
| | 9. ~K ∨ ~N | 8, Add |
| | 10. ~(K • N) | 9, DM |
| | 11. M ∨ L | 2, 10, MP |
| | 12. L | 7, 11, DS |
| | 13. ~L • L | 6, 12, Conj |
| 14. ~ ~(K ∨ L) | | 1–13, IP |
| 15. K ∨ L | | 14, DN |

QED

7.

| | | |
|---|---|---|
| 1. A ⊃ B | | |
| 2. ~C ⊃ ~(A ∨ ~D) | | |
| 3. ~D ∨ (B • C) | | / A ⊃ (B • C) |
| \| 4. A | | ACP |
| \| | 5. ~(B • C) | AIP |
| \| | 6. ~B ∨ ~C | 5, DM |
| \| | 7. B ⊃ ~C | 6, Impl |
| \| | 8. B ⊃ ~(A ∨ ~D) | 2, 7, HS |
| \| | 9. A ⊃ ~(A ∨ ~D) | 1, 8, HS |
| \| | 10. ~(A ∨ ~D) | 4, 9, MP |
| \| | 11. ~A • ~ ~D | 10, DM |
| \| | 12. ~ ~D • ~A | 11, Com |
| \| | 13. ~ ~D | 12, Simp |
| \| | 14. B • C | 3, 13, DS |
| \| | 15. ~(B • C) • (B • C) | 5, 14, Conj |
| \| 16. ~ ~(B • C) | | 5–15, IP |
| \| 17. B • C | | 16, DN |
| 18. A ⊃ (B • C) | | 4–17, CP |

QED

8.

| | | |
|---|---|---|
| 1. ~[J ∨ (F • ~H)] | | |
| 2. ~G ⊃ ~H | | |
| 3. G ∨ [~F ⊃ (J • K)] | | / E ∨ G |
| 4. ~J • ~(F • ~H) | | 1, DM |
| 5. ~(F • ~H) • ~J | | 4, Com |
| 6. ~(F • ~H) | | 5, Simp |
| 7. ~F ∨ ~ ~H | | 6, DM |
| 8. ~F ∨ H | | 7, DN |
| \| 9. ~(E ∨ G) | | AIP |
| \| 10. ~E • ~G | | 9, DM |
| \| 11. ~G • ~E | | 10, Com |
| \| 12. ~G | | 11, Simp |
| \| 13. ~F ⊃ (J • K) | | 3, 12, DS |
| \| 14. ~H | | 2, 12, MP |
| \| 15. H ∨ ~F | | 8, Com |
| \| 16. ~F | | 15, 14, DS |
| \| 17. J • K | | 13, 16, MP |
| \| 18. ~J | | 4, Simp |
| \| 19. J | | 17, Simp |
| \| 20. J • ~J | | 19, 18, Conj |
| 21. ~ ~(E ∨ G) | | 9–20, IP |
| 22. E ∨ G | | 21, DN |

H)

1. ~Sa • (La • Sa)
2. Ac ⊃ (~Ab • Fb)
3. ~Ae ⊃ (~We v Le)

I)

1.

| | | |
|---|---|---|
| 1. | (∀x)Ax ⊃ Ba | |
| 2. | (∀x) ~(Ax ⊃ Cx) | / (∃x)Bx |
| 3. | ~(Ax ⊃ Cx) | 2, UI |
| 4. | ~(~Ax ∨ Cx) | 3, Impl |
| 5. | ~ ~Ax • ~Cx | 4, DM |
| 6. | ~ ~Ax | 5, Simp |
| 7. | Ax | 6, DN |
| 8. | (∀x)Ax | 7, UG |
| 9. | Ba | 1, 8, MP |
| 10. | (∃x)Bx | 9, EG |

QED

2.

| | | |
|---|---|---|
| 1. | (∃x)(Dx • Fx) | |
| 2. | (∃x)(Gx ⊃ Ex) | |
| 3. | (∀x) ~(Hx ∨ Ex) | / (∃x)Fx • (∃x) ~Gx |
| 4. | Da • Fa | 1, EI |
| 5. | Gb ⊃ Eb | 2, EI |
| 6. | ~(Hb ∨ Eb) | 3, UI |
| 7. | ~Hb • ~Eb | 6, DM |
| 8. | ~Eb • ~Hb | 7, Com |
| 9. | ~Eb | 8, Simp |
| 10. | ~Gb | 5, 9, MT |
| 11. | Fa • Da | 4, Com |
| 12. | Fa | 11, Simp |
| 13. | (∃x)Fx | 12, EG |
| 14. | (∃x) ~Gx | 10, EG |
| 15. | (∃x)Fx • (∃x) ~Gx | 13, 14, Conj |

3.

| | | |
|---|---|---|
| 1. | (∀x)(Lx ⊃ ~Nx) • (∀x)(~Mx ⊃ ~Ox) | |
| 2. | (∀x) ~(~Nx • ~Ox) | / (∀x)(Lx ⊃ Mx) |
| 3. | (∀x)(~ ~Nx ∨ ~ ~Ox) | 2, DM |
| 4. | (∀x)(~ ~Nx ∨ Ox) | 3, DN |
| 5. | (∀x)(~Nx ⊃ Ox) | 4, Impl |
| 6. | ~Ny ⊃ Oy | 5, UI |
| 7. | (∀x)(Lx ⊃ ~Nx) | 1, Simp |
| 8. | Ly ⊃ ~Ny | 7, UI |
| 9. | Ly ⊃ Oy | 8, 6, HS |
| 10. | (∀x)(~Mx ⊃ ~Ox) • (∀x)(Lx ⊃ ~Nx) | 1, Com |
| 11. | (∀x)(~Mx ⊃ ~Ox) | 10, Simp |
| 12. | ~My ⊃ ~Oy | 11, UI |
| 13. | Oy ⊃ My | 12, Cont |
| 14. | Ly ⊃ My | 9, 13, HS |
| 15. | (∀x)(Lx ⊃ Mx) | 14, UG |